ME-GY 7943

Network Robotics Systems, Cooperative Control and Swarming

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Exercise series 2

Exercise 1

**1)** **Prove that is a constant of motion. How can you interpret the result ?**

**Solution:**

To prove that is constant of motion

we also know that **,** Substitute in above equation,

Therefore,

**=** 0

As  **=** 0 we can tell that if the system starts at a point it will converge at that point.

**2)** From Figure 1. Only graph b will converge because it a rooted-out branching and all other graph have cycle and are not rooted out branching. Yes the Convergence point will depend on the initial condition.

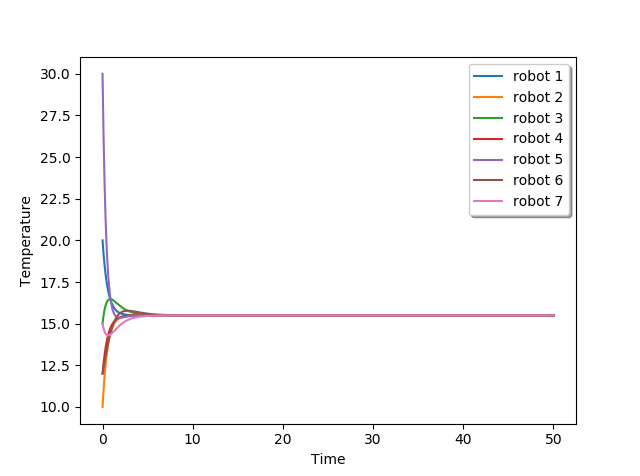
**3)** The graphs that are converging are graph a, b and d.

The converging point for graph a is 15.5.

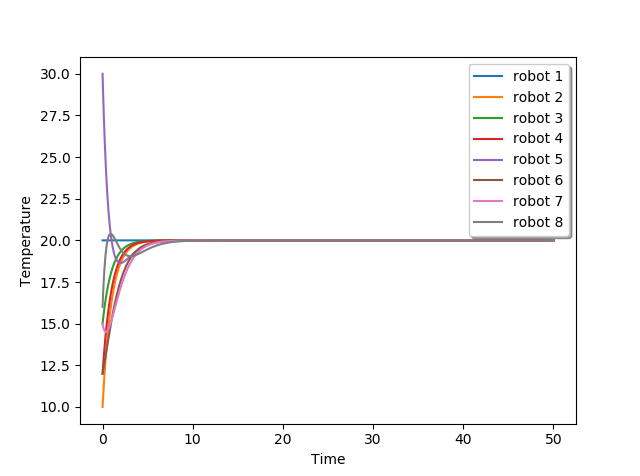
The converging point for graph b is 20.

The converging point for graph d is 16.249.

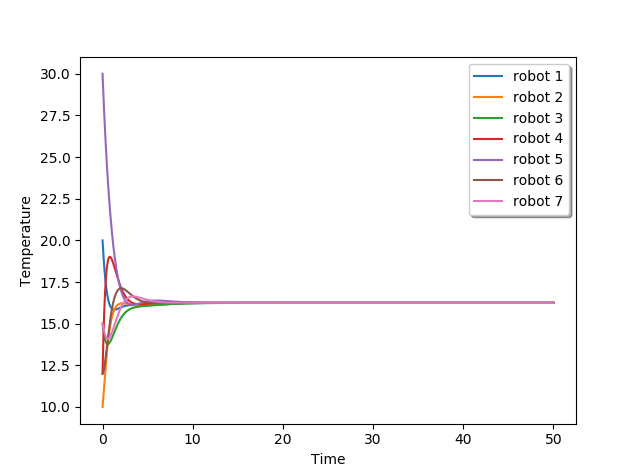
**4)** Below shows the behavior for the graphs a, b and c. The code for each graph is included in the file exercise1\_4 .

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**Fig 1** Graph a for figure 1

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**Fig 2.** Graph b for figure 1

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**Fig 3.** Graph d for figure 1

**Exercise 2**

**1)**

To find the fixed point

So we get, and

So the only fixed point we get to satisfy the above condition is and .

**2)**

**a)** Yes, + is a Lyapunov function. Below shows the proof.

Given dynamical system,

+

+

=

So from the above equation we can see that as for any values of x and y the function is negative. Since the system admits Lyapunov function it is asymptotically stable.

**b)**

After finding the derivative of the we get . So since putting any values of the x we get positive it does not satisfy . Hence its not Lyapunov function.

**c)**

+

Since , it’s a Lyapunov function.

**Exercise 4**

**1)** Yes, for each graph in figure 2 the consensus protocol will converge because each graph is rooted-out branching.

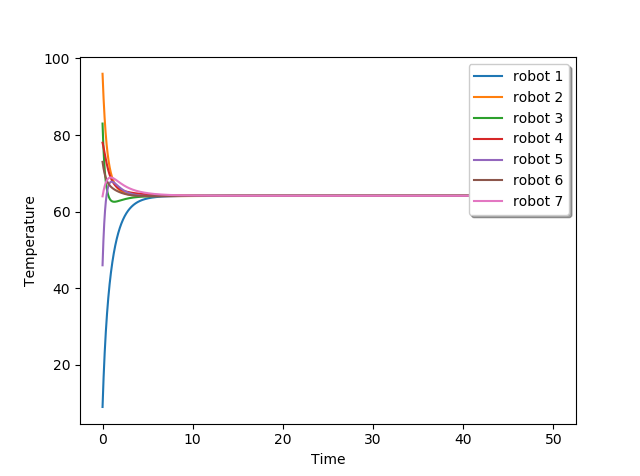
**2)** Yes, the system with switching graph will converge because since each graph is rooted-out branching , superimposing each other or union of all graph still gives rooted out branching. Therefore in figure 1 switching graphs will converge.

**3)** The code is in Ex4.3.py file.

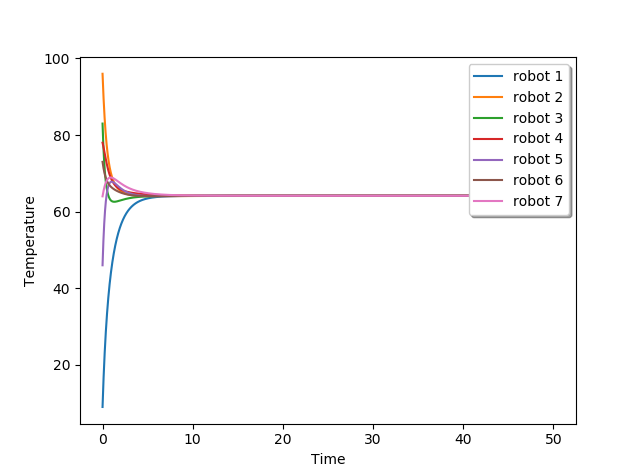
**4)** Below shows the behavior for graph switching every 2sec and 0.1sec respectively.

Using initial condition as x\_0 = [9, 96, 83, 78, 46, 73, 64] . The graph below shows convergence for

Switch time 0.1 and 2 seconds and both the graph converge at same point because the graphs are balanced and rooted- out.

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**Fig 4** Graph switching every 0.1 sec



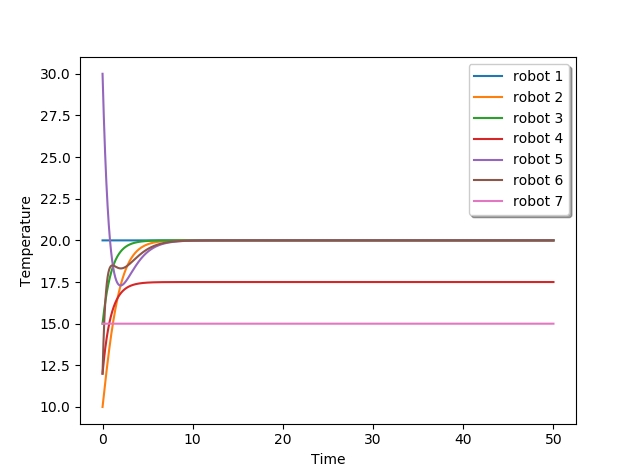
**Fig 5** Graph switching every 2 s

**Exercise 5**

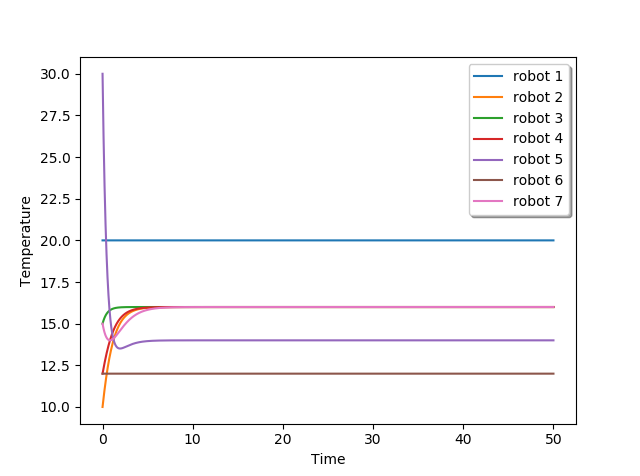
**1)** No, the consensus protocol will not converge for any graph because there is no rooted out branching.

**2)** Yes, Because superimposing the graphs will give rooted out branching.

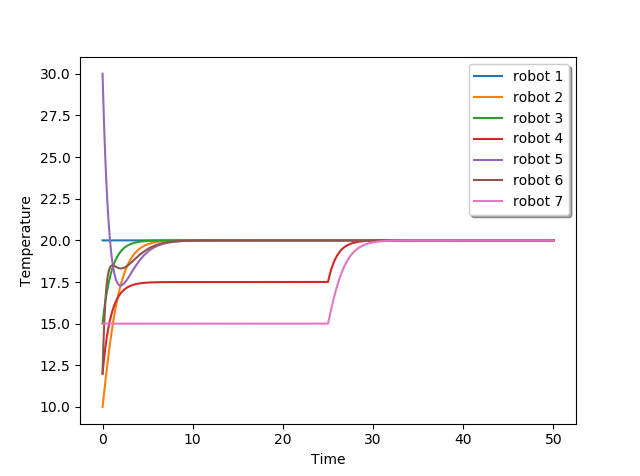
**3)** Graph below shows the behavior for each graph in figure 3 separately and with switching .



**Fig 6** Graph for figure 3a

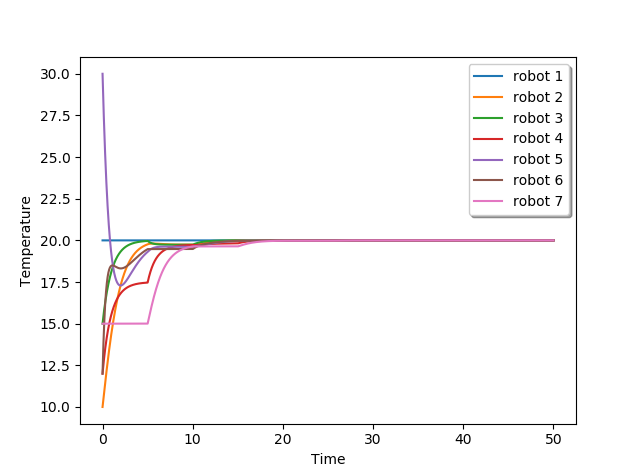


**Fig 7** Graph for figure 3b

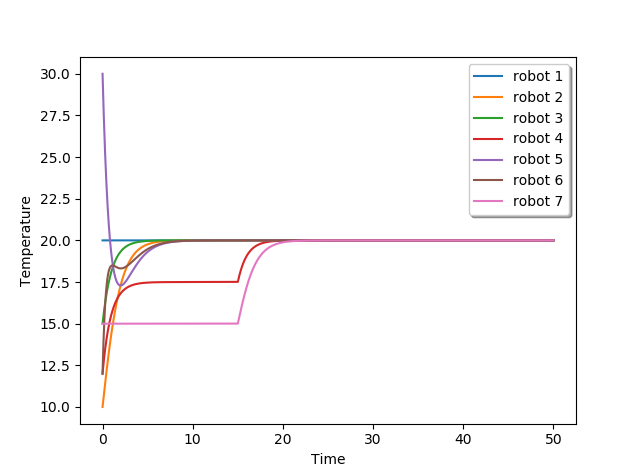


**Fig 8** Switch time 0.5 (figure 3)

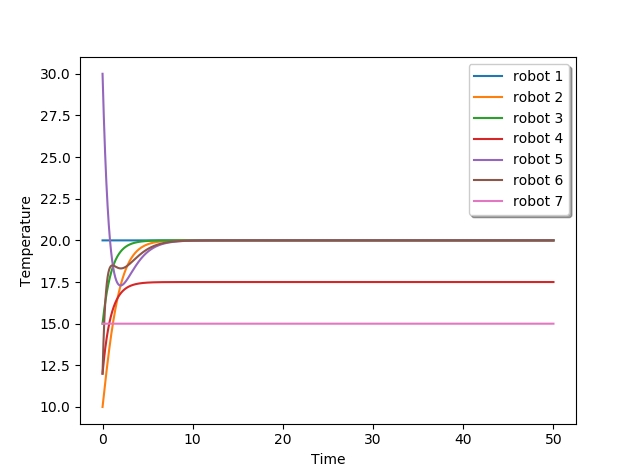
**4)** Below are the switching graph every 0.1 , 0.3 and 2 seconds respectively for figure 3.



**Fig 9** Switching for graph 3 at 0.1 sec



**Fig 10** Switching for graph 3 at 0.3 sec



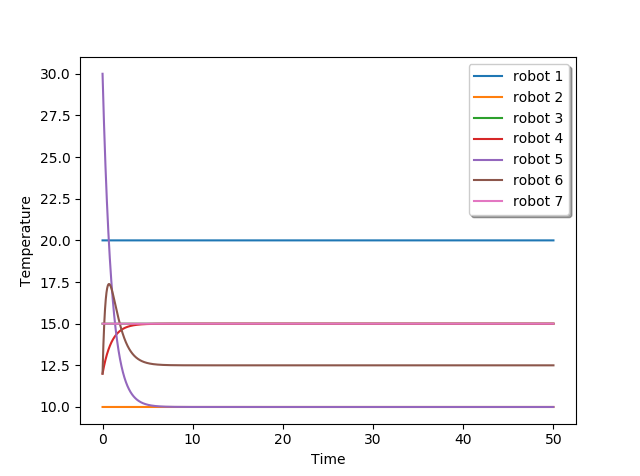
**Fig 11** Switching for graph 3 at 2 sec

**Exercise 5 (Q5) – Figure 4**

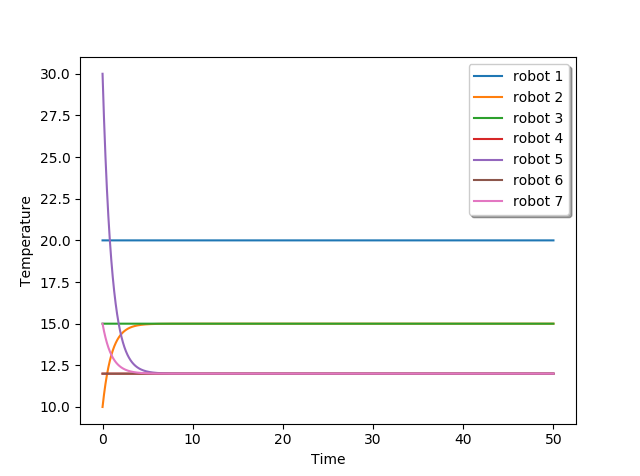
**1)** No, the consensus protocol will not converge as there is not rooted out branching in any graph for figure 4.

**2)** Yes, Because superimposing the graphs will give rooted out branching.

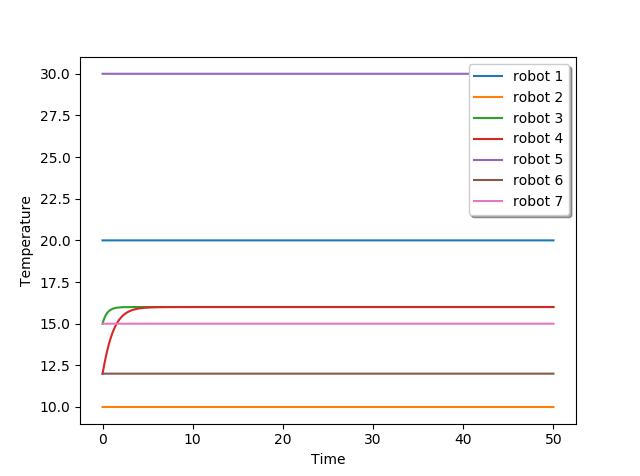
**3)** Consensus protocol for each graph in figure 4 separately and with switching happening every 0.5 sec.



**Fig 12** Graph for figure 4a



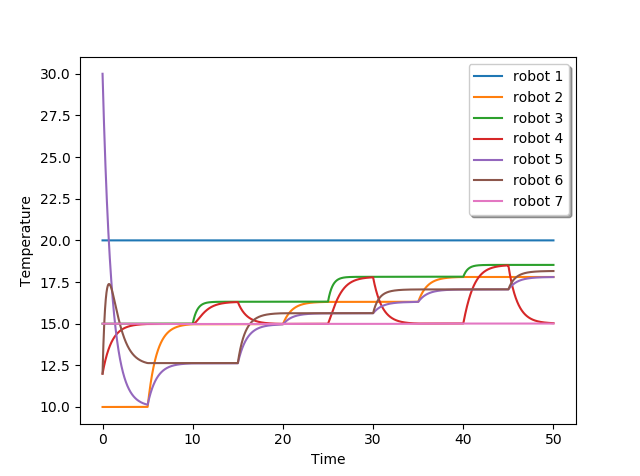
**Fig 13** Graph for figure 4b



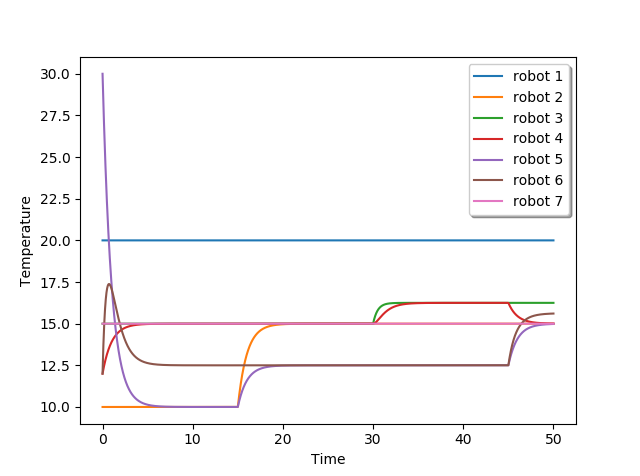
**Fig 14** Graph for figure 4c

**4)** Below are the graph for figure 4 for switch time 0.1, 0.3 and 2 seconds respectively.

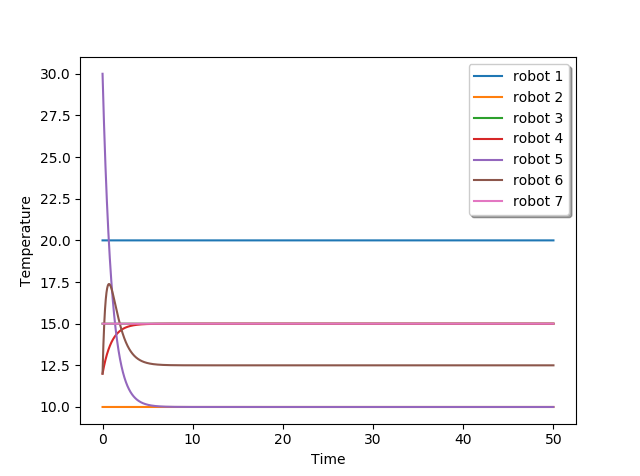
The graph does not converge as shown below for T=50 sec.



**Fig 15** Graph for figure 4 for switch time 0.1 sec

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**Fig 16** Graph for figure 4 for switch time 0.3 sec



**Fig 17** Graph for figure 4 for switch time 2 sec